Synopsis

on

STUDY OF SOME BIOLOGICAL MODELS USING INTERVAL NUMBER AND FUZZY SETS

by

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A thesis submitted in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy
(Mathematics)

under the supervision of
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2025

SYNOPSIS

This presentation is mainly concerned with some biological models formulation and model analysis using Interval number and Fuzzy sets. In this thesis we proposed various bio-mathematical model with co-efficients are Interval numbers or L-R Fuzzy number. Here we find solution of proposed biological model and cheek positivity , boundedness. In this thesis we examine all type of stability analysis at every equilibrium points. Routh-Huriwitz criteria is more important tool for scrutinize the stability of the model.

Mathematical preliminaries are concerned with general introduction , basic definition , some lemma , various type of differentiation such as Hukuhara derivative, Fractional derivative and mathematical technique which is used to the analytical and numerical solution of fuzzy non linear differential equation and fractional differentiation discussed in this thesis.

The contents of the presentation into three parts first part contain Mathematical preliminaries second contain model formulation and last part contain numerical simulation and graphical presentation.

Actually, biological model transfer to system of non linear fuzzy differential equation. Hukuhara differentiation is most important for my work. We investigated stability analysis when all or some of parameters are (i)-gH differentiable or (ii)-gH differentiable or both.

In this thesis our first proposed biological model for diabetes and pre- diabetes in fuzzy environment. Due to the presence of fuzzy variable, the model is transformed to a fuzzy differential equation. Here we can use the concept of fuzzy differential equation for solving and stable analysis of the model and after comparing the all kind solutions of fuzzy and crisp solution of diabetes and pre-diabetes model. The strong point of our mathematical model is biological meaningfull. The conception of strong fuzzy solution and weak solution of fuzzy diabetes and prediabetes model are discussed and suggested which is relevant to this model. Stability criteria have been compared in both cases when fuzzy functions are (i)-gH differentiable and when fuzzy functions are (ii)-gH differentiable. The fractional order SI compartmental model with saturated treatment rate is explored in this manuscript with three different categories in the Caputo approach. A few conclusion have been made about the existence of the new model, the uniqueness requirement, non-negativity and boundedness of the solution. The system is locally and globally asymptotically stable at the disease free equilibrium point E_0 and the epidemic equilibrium E_1, according to the models stability analysis.The all results are verified graphically and numerically using MATLAB software.

The modern mathematical model is an interpretation of the mathematical description of various global ecological issues. Over the past few decades, ecologists and researchers have shown the greatest interest in the modelling of prey-predator kinetics. The most of researchers in ecology

have assumed that ecological parameters are exactly known. Indeed, the actual situation is altered when the uncertainty in the model may occur due to several cause like human error, inaccurate data supply, climate changes and different environmental factors etc. To overcome such situation we consider the Lotka–volterra harvesting model in which initial number of populations are assumed as fuzzy number. To elucidate the fuzzy proposed system, we have use a methodology which is gH (generalised Hukuhara) derivatives concept. When this technique is applied to the fuzzy prey-predator system, the leading model is transformed into a set of differential equation with parametric form of α.Here we consider two cases only, both the prey population($v_1(t)$) and predator population($v_2(t)$) be (i)gH,(ii)-gH differentiable. Non-negative feasible steady states and stability criteria of them have been analyzed in fuzzy environment. Strong and weak solution of the proposed system have been evaluated. In the end, extensive numerical simulations verify all of the analytical findings.

FDE i.e Fuzzy Differential Equation has become highly acceptable tool to analyze the various kind of mathematical model from diverse field of science, engineering, technology and management. Due to some sort of environmental or demographic vagueness the differential equation or model does not behave like classical i.e crisp differential equation. In this situation to handle the behavior of the system, scientists and researchers have been improving the theory of FDE. When some parameters or co-efficient, initial or boundary condition(s), suitable forcing term or all possible combination these are taken account of fuzzy sense, the differential equation is called FDE(Fuzzy Differential Equation). During last few year FDE has been developed rapidly after introducing numerous strategy for solving FDE. Base on different technique/ method (Zadeh's Extension principal, fuzzy differential inclusion, fuzzy function differentiability concept and fuzzy bunch of real valued functions instead of fuzzy valued functions). At first Chang and Zadeh [3] introduced the concept derivative of fuzzy function. Pittschmann et al[4] and Buckley et al [5] analyzed FDE i.e fuzzy differential equation using extension principle.

In this article, the thoughts and idea are deal with as follows

(i) A pre diabetes and diabetes system is with non-homogeneous system of linear differential equation and fuzzy environment.

(ii) When the pre diabetes and diabetes model a fuzzy the pre diabetes and diabetes model .converted into corresponding crisp model is discussed.

(iii) Due to presence of fuzzy variable the fuzzy the pre diabetes and diabetes model is transformed different cases is illustrated.

(iv)How the fuzzy the pre diabetes and diabetes model reduced to crisp system of equation in parametric form by characterization theorem is discussed.

(v) The kinetics of proposed model has been compared between both environment (fuzzy and crisp).

(vi) The conception of fuzzy solution (strong and weak solution) has been compared numerically and verified by results.

The improvement of the article will confidently assistance researchers who work, modeling biology as well as fuzzy mathematic and fuzzy differential equation. Using this perceptions researcher can straightforwardly determine the solution and study the stability analysis on any real.

Mathematical modeling is a crucial and required technique for addressing a wide range of real-world issues. In several fields of science and technology, Fuzzy Differentiation Equation is play very important role in Agriculture and Medical science. Many mathematical modeling such as Diabetes model, Dengue model, HIV model, SI model and many other epidemic disease model all models express by Fuzzy Differential Equation(FDE). Pest control is important mater in agriculture. Pest control is depends prey-predator systems that can be express in mathematical modeling which is the simultaneous fuzzy differential equation.

Mathematical modeling is a vital and necessary tool for dealing with a variety of real problems. In several fields of science and technology, researchers have given close attention to modeling on various types of phenomena. Utilizing mathematical modeling, some elements of biological science and ecology have been quickly and attractively captured and described. Interfaces and interaction across biotic and abiotic components have been added up through various functions in theoretical ecology and ecological modeling, where many essential parameters have been employed. Researchers were being provided information to represent real-life occurrences by constructing various types of mathematical models in an imperfect environment during the previous few decades. The dynamical models of such interacting species are provided by Lotka-Volterra systems of the equations and there are three types of model equations dealing with interactions [1-8]. Three models are (i) predator-prey model (ii) competition model (iii) mutualistic model. The characteristics of the single spices population in which like organisms associates to includes population growth and reproductions, regulation and equilibrium and fluctuation and periodicity [9-18]. Such types of phenomena are the very special topic of study of the population ecologist, as the effect of the chemical and physical factor are the domain of the physiological ecologists. In reality any ecosystem consists of several species which are interrelated among them. Actually multi-species population models to understand the nature and diversity of natural ecosystem. So there are different types of interactions between two different species living in the same ecosystem.

The aim of mathematical modeling is to imitate some real-world problems as far as possible. The presence of imprecise variable and parameters in practical problems in the field of bio-mathematical modeling became a new area of research in uncertainty modeling. So, the solution procedure of such problems is very important. If the solution of said problems with uncertainty is developed then, many real life models in different fields with imprecise variable can be formulated and solved easily and accurately.

The ecological models using fixed biological parameter values with the help of linear or nonlinear system of ordinary differential equations are formulated in precise environment. Those models may be far away from reality of ecological systems. We have considered prey-predator population model in uncertain environment to imply these realities in our work. Though the uncertainties may be both stochastic and non stochastic sense but uncertainty in stochastic sense is more difficult to analyses and constructs a suitable probability distribution function. Therefore we consider the uncertainty in non-stochastic sense which is interval. The population density, intrinsic growth rate, harvesting rate, predation rate etc, in ecological dynamics are usually uncertain. This encouraged us to look for innovative ecological models and find out the more practical justification.

An interval number is itself a imprecise parameter. Because the value is not an crisp number, the value is lie between two crisp number. When we take any parameter may be coefficients or initial condition or both of an differential equation then the interval differential equation comes. The basic behaviors of that number are different from a crisp number. So the calculus of that types numbers valued functions are different. So we need to study the differential equation in these environments.

When we formulate models with real-life problems, some data are not precise. In such situations if we consider precise real-life data there are chances of error. So, we need to take the parameter as an imprecise. In such cases a question may arise, whether the behavior of the model is same as a crisp model. The Behavior is different obviously. We need to investigate the behavior in that environment. In our present paper we modeled with differential equation with in imprecise environment namely fuzzy environment. Now we considered the different cases of fuzzy environment. We need to solve the problems using fuzzy derivative concepts because the fuzzy derivative is not same as crisp derivative. Also we need to discuss the behavior of the model in fuzzy environment.

In the area of theoretical ecology the research was started by [1, 2] Lotka and Volterra. Though the first theoretical management of population dynamics was presented by [3, 4] Malthus and Verhulst formed a mathematical model based on logistic equation. The role of Lotka-Volterra model is intervention in competition model, i.e. two species are assumed to diminish each other's per capita growth rate by direct intervention.The Lotka-Volterra model of interspecific competition has been a useful starting point for biologists with the thought about the outcomes of competitive interactions between species. The assumptions of this model (e.g., there can be no migration and the carrying capacities and competition coefficients for both species are constants) may not be very realistic, but are necessary for simplifications. A variety of factors not included in the model can affect the outcome of competitive interactions by affecting the dynamics of one or both populations. Environmental change, disease, and chance are just a few of these factors.

Mathematical models of population growth have been formed [5] to provide an impalpable of some significant aspect of true ecological situation. Here the meaning of each parameter in the

models has been defined biologically. Verhust first proposed three-parameter model for the growth of single-species populations that shows a logistic sigmoid growth curve for time. Pearl and reed independently expressed the same model, which cover mathematically an upper asymptote and a rate constant. The increasing study of realistic and effictively useful mathematical models in population biology, where we are dealing with a human population with or without its age distribution, population of an endangered species, bacterial or viral growth and so on,is a reflection of their use in helping to understand the dynamic process involved and in making practical prediction.

Differential equations may arise in the mathematical modeling of real-world problems. But when the impreciseness comes to it, the behavior of the differential equation is altered. The solution procedures are taken in different way. In this paper we take two types of imprecise environments: Fuzzy and Interval and find their exact solution. In 1965, Lotfi A. Zadeh [6] published the first of his papers on the new theory of Fuzzy Sets and Systems. After that Chang and Zadeh [7] introduced the concept of fuzzy numbers. Last few years researcher give their great contribution on the topic of fuzzy number research [8-10]. As for the application of the fuzzy set theory applied in fuzzy equation [11], fuzzy differential equation [12], fuzzy integro-differential equation [13-15], fuzzy integral equation [16] etc. were developed. An interval number is itself an imprecise parameter. Because the value is not a crisps number, the value is lie between two crisp numbers. When we take any parameter may be coefficients or initial condition or both of a differential equation then the interval differential equation comes. The basic behaviors of that number are different from a crisp number. Hence, the calculus of those types numbers valued functions is different. So we need to study the differential equation in these environments. From the time that Moore [42] and Markov [43] as the pioneers introduced the interval analysis and related notions, several monographs and papers were devoted to connect the fuzzy analysis and interval analysis [44], but, the later one was not well-realized and applicable to model dynamical systems. After introducing generalized Hukuhara-differentiability, different perspectives, which leaded to nice schemes and strategies to achieve the solutions, were discussed in the literature [45-48]. Lupulescu in [49] developed the notions of RL- and Caputo-types derivatives for interval-valued functions. Salahshour et al. [50, 51] proposed a non-singular kernel and conformable fractional derivative for interval differential equations of fractional order. Recently interval differential equation is studied by Costa et al.

Using ordinary differential equation we can write the kinetic of some diabetes model[39-42]. There exist different models where researcher gives their contribution for modeling a diabetes. Here we give an overview of some model as [43] studied Generalized Mathematical models on glucose insulin kinetics in IVGTT by considering special cases by Delay differential equation (DDE). Gaetano et al [44, 45] and Li and Kuang [46] formed more realistic models of glucose insulin dynamics introducing delay integro-differential equation. Wach et. al. [47] implemented Diabetes modeling on concentration of insulin in tissuein both form (dimeric and hexameric) applying PDE(Partial differential equation) ,moreover by the help of significant mathematical

tool SDE(stochastic differential Equation) a short term prediction on glucose regulatory system based on GB(Grey box) model has been smeared in[48] and [49] studied a Physiological model of type 1 diabetes patients in stochastic environment using parameter estimation. During the formation of real world problems using mathematical modeling, just because of some information are not known precisely, data are collected imprecisely, error in data supply by human, Instrument, computer eventually environmental effect the uncertainty or vagueness[19,35,50-53] comes into the model .In that case the kinetics of model become more realistic than its previous crisp or precise form. So we need to investigate the behavior of model on diabetes in imprecise environment. [54] Investigated the kinetics of diabetes model in fuzzy environment and solved it via FDE approach using generalized H-differentiability concept. The nature of fuzzy solution (weak and strong fuzzy solution) is most important to understand the dynamics of model. A strong point about weak fuzzy solution has been proposed by A. Ahamidian et.al [55].Now we need to take into account a model on pre diabetes and diabetes [16] to establish a comparison a behavior of the pre- diabetes and diabetes system in fuzzy and crisp environment.

An essential tool for describing biological and physical processes is fractional calculus. Fractional calculus has been used recently in a wide range of scientific fields, either theoretically or practically. For instance, it is used to study the conservative and non-conservative systems using fractional differential equations[], and to characteristics the non-differentiable points of continuous functions. Thus, it may be said that although fractional calculus is developing slowly, it will eventually become an essential instrument in science.

Lotka[84] and Volterra[85] initiated research in the field of ecology. Although Malthus [86] presented the first theoretic managing of population kinetics, Verhulst [87] developed a logistic equation-based model. In the Lotka-Volterra system, direct intervention is supposed to reduce the per capita growth rates of two species. For biologists who are interested in the consequences of competitive interactions between species, Lotka-Volterra equation of Recent time, the“fuzzy differential equation[93-196]" (FDE) has become increasingly popular. Henceforth, several notable work of bio-mathematical modeling has been published with based on fuzzy differential equation(see[97-101]).Here, motivated by the above theme we have considered Lotka –Volterra with harvesting system[89] in fuzzy environment.

exploitative competition has served as a suitable initial point. This model's presumptions may not be very realistic, but they are necessary for simplification. The dynamics of one or both populations can be affected by a variety of non-model factors that affect the outcome of competitive interactions. Changes in the environment, illness, and chance are just some of these factors. Over the past few decades, ecologists and researchers have shown the greatest interest to formulate different types harvested [88-91] (prey or predator or both harvested) model to explore better dynamics of ecological phenomena into models.

In the field of ecological science, many authors have evolved their system constructed totally on the presumption that the model parameters are familiar. However indeed, the parameters of model system are not exact because of inaccuracy in data collection, unconsciously measurement, and technical error and climate changes. To overcome such type to complexity many authors use different approach –FDE(Fuzzy differential approach),IDE(Interval differential equation approach)and Stochastic approach[92] etc.

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The arrangement of the chapter follows as Sec 3.2 contains the Lotka-Volterra Prey-Predator Harvested Model is express by first order differential equation with initial condition. The first order non homogeneous fuzzy differential equation is discussed in four different cases in Sec3.2.2 and check stability analysis in various equilibrium points. Numerical illustration and graphical presentation is prepared in Sec3.3. overall conclusion are drawn in section 3.4.

Organisation of the thesis:

The thesis is organized as follows:

We have divided our entire proposed thesis into the following divisions and chapters.

Part-I: Introduction, Basic Concepts and Preliminaries.

Chapter 1: Introduction

Chapter 2: Basic Concepts

Part-II:

Chapter 3: Study of Fuzzy Prey Predator Harvested Model: Generalised Hukuhara Derivative Approach

Chapter 4: Study of Prey Predator Model Formulation and Stability Analysis

Chapter 5: Comparison between pre-diabetes and diabetes model in fuzzy and crisp environment: fuzzy differential equation approach:

Chapter 6: SOME DIABETES MODEL IN FUZZY ENVIRONMENT: REVISIT

Chapter 7: Study of dengue model with temperature effects under interval uncertainty

Chapter 8: Study of Fractional order SI model with Stability analysis

Part-III: Summary of the thesis and future research scope

In Chapter 1, a brief introduction that consists of historical background and motivation about research works have been described. In Chapter 2, mathematical preliminaries have been explored. In Chapter 3, we formulate and discuss of Fuzzy Prey Predator Harvested Model: Generalised Hukuhara Derivative Approach. In Chapter 4 ,we formulate and stability analysed of Prey Predator Model . In Chapter 5, compared between pre-diabetes and diabetes model in fuzzy and crisp environment under fuzzy differential equation approach.
In Chapter 6,revisit some diabetes model in fuzzy environment. In this chapter, we have reviewed several of our previously published models. The 'glucose insulin regulatory system' in Model 1 is an IVP whose initial values are physiological. We have elucidated this model by utilizing the Hukuhara derivative and presuming the initial value as fuzzy number. The fuzzy diabetes model, which is described in Model 2, was derived from the paper by Boutayeb et al. The solution is derived using the generalized Hukuhara derivative in Model 3, using the fundamental model of "Diffusion of glucose and drug in blood stream in fuzzy environment" and employing biological initial conditions. I have conducted an analysis of the stability of its equilibrium points by utilizing the eigen values of the variational matrix.

The system on Diabetes and Pre-diabetes models in imprecise environments is revisited in Model 4. I verified its stability by examining the results at various interior sites. Additionally, we engage in an intelligent discussion regarding its restrictionsIn Chapter 7, under interval uncertainty , dengue model with temperature effects . Here the population is divided into three groups: pre-adult vectors, adult female mosquitoes, and human hosts. For the A. aegypti pre-adult female mosquito population, two parameters—eggs and larvae—are defined within both the susceptible (Se) and infected (Ie) populations. S_v, E_v, and I_v, which stand for the number of susceptible, infected but not yet infectious, and infectious female mosquitoes at time t, respectively, were specified as the three parameters for the adult vector (mosquito) population. Similarly, three parameters were found for the host population (humans)ere In Chapter 8: Study of Fractional order SI model with Stability analysis.

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